

<b>Noticing Linear Patterns</b>	For each pattern below, identify how the terms change from one to the next.	
	1, 2, 3, 4, 5, 6, ... $+1 +1 +1$ $(+1)$	3, 5, 7, 9, 11, 13, ... $(+2)$
	12, 22, 32, 42, 52, 62, ... $(+10)$	50, 40, 30, 20, 10, 0, ... $(-10)$
	4, 1, -2, -5, -8, -11, ... $(-3)$	2, 0.5, -1, -2.5, -4, -6.5, ... $-3/2$ $(-1.5)$

**Arithmetic Sequence (Linear)**  
Since a number is added or subtracted to get the next in each pattern above, they are called Arithmetic Sequences.

**Common Difference**  
The number that is added or subtracted is called the "common difference" as it can be found by subtracting a term and the previous term (the one right before).

**Example**  
A gardener buys a plant that is 12 cm in height. Each week after that the plant grows 5 cm.

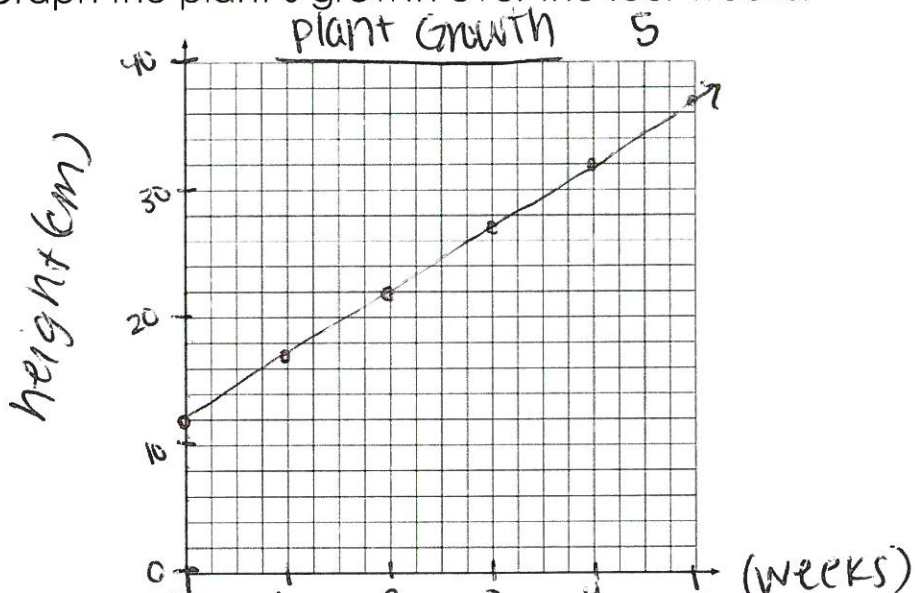
What will the height of the plant be after the first week? Second, third, fourth, and fifth weeks? Fill in the table to help you.

Week	0	1	2	3	4	5
Height	12	17	22	27	32	37

$+5$     $+5$     $+5$     $+5$     $+5$

What is the initial value?  
12cm  $\swarrow$  y-intercept

What is the common difference?  
5cm  $\swarrow$  slope

<p><b>Example continued</b></p>	<p>Graph the plant's growth over the <del>four</del> weeks.</p>  <p>Write the linear equation for the plant's growth:</p> $y = mx + b$ $y = 5x + 12$
<p><b>Linear Connection</b></p>	<p>The equation above represents the "Explicit Formula" for the sequence of heights of the plant. For an arithmetic sequence, the explicit formula can be written in <math>y = mx + b</math> form.</p>
<p><b>Explicit Formula</b></p>	$f(n) = dn + f(0)$ <p>(Slope) <math>d = \text{common difference}</math> <math>n = \text{input (x value)}</math> <math>f(0) = \text{starting value}</math> (y-int)</p> <p><math>f(n) = \text{output (y value)}</math></p>
<p><b>f is for function</b></p>	<p>The <math>f</math> in the formula above represents "function." It is a notation used to show that the <math>f(n)</math> value depends on the <math>n</math> value. This is exactly how <math>x</math> and <math>y</math> normally work together in a linear equation.</p> <p>Generally, you can think of <math>f(n)</math> as the <math>y</math>-value (output), and <math>n</math> as the <math>x</math>-value (input).</p>
<p><b>Using the Common Difference</b></p>	<p>When we filled in the table for the plant's height, we were using what is called the Recursive Rule, or Recursive Formula. The Recursive Rule is where you use the common difference and a term to find the next term in the sequence.</p>

**Recursive Formula**

$$f(n) = f(n-1) + d; f(0) = a \quad n = \text{term \#}$$

$f(n)$  = current term's value     $f(n-1)$  = previous term's value     $d$  = common difference     $f(0) = a$  starting value

**Example**

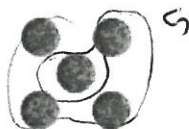
For the plant's height, the recursive formula would be:

$$f(n) = f(n-1) + 5; \quad f(0) = 12$$

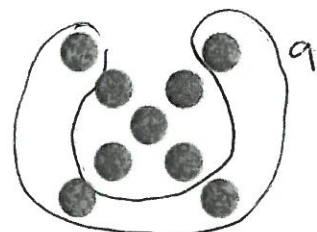
**Example**



At the beginning



At one minute



At two minutes

**For the pattern above, complete the following:**

Make a table for the values in the pattern:

$n$	0	1	2
$f(n)$	1	5	9

+4

+4

Common Difference:

$$d = 4 \text{ dots}$$

Initial Value:

$$f(0) = 1 \text{ dot}$$

Explicit Equation:  $f(n) = dn + f(0)$

$$f(n) = 4n + 1$$

Recursive Equation:  $f(n) = f(n-1) + d; f(0) = a$

$$f(n) = f(n-1) + 4; \quad f(0) = 1$$

