

<b>Multiplying &amp; Dividing Patterns</b>	For each pattern below, identify how the terms change from one to the next.	
	1, 2, 4, 8, 16, ... $\cdot 2 \cdot 2 \cdot 2 \cdot 2$ $\frac{8}{4} = \cdot 2$	1, 3, 9, 27, 81, ... $\cdot 3$
	1, 10, 100, 1000, 10000, ... $\cdot 10$	3, 6, 12, 24, 48, ... $\cdot 2$
	80, 40, 20, 10, 5, 2.5, 1.25, ... $\div 2$ $\cdot \frac{1}{2}$ $\cdot 0.5$	4, 2, 1, $\frac{1}{2}$ , $\frac{1}{4}$ , ... $\div 2$ $\cdot \frac{1}{2}$ $\cdot 0.5$

**Geometric Sequence** Since a number is multiplied or divided to get the next number in the sequence in each pattern above, they are called Geometric Sequences.  $\cdot$  or  $\div$

**Common Ratio** The number that is multiplied (the reciprocal number divided) is called the "common ratio" as it can be found by dividing a term by the previous term (the one right before it).  
 $\frac{f(n)}{f(n-1)}$

**Example** A scientist is studying bacteria that doubles in number every day. If he starts with 50 cells of bacteria, how many cells will there be after one day? Two days? Three, four, five days? Fill in the table to help you.  $\times 2$

<b>Day</b>	0	1	2	3	4	5
<b>Cells</b>	50	100	200	400	800	1600

$\times 2$     $\times 2$     $\times 2$     $\times 2$     $\times 2$

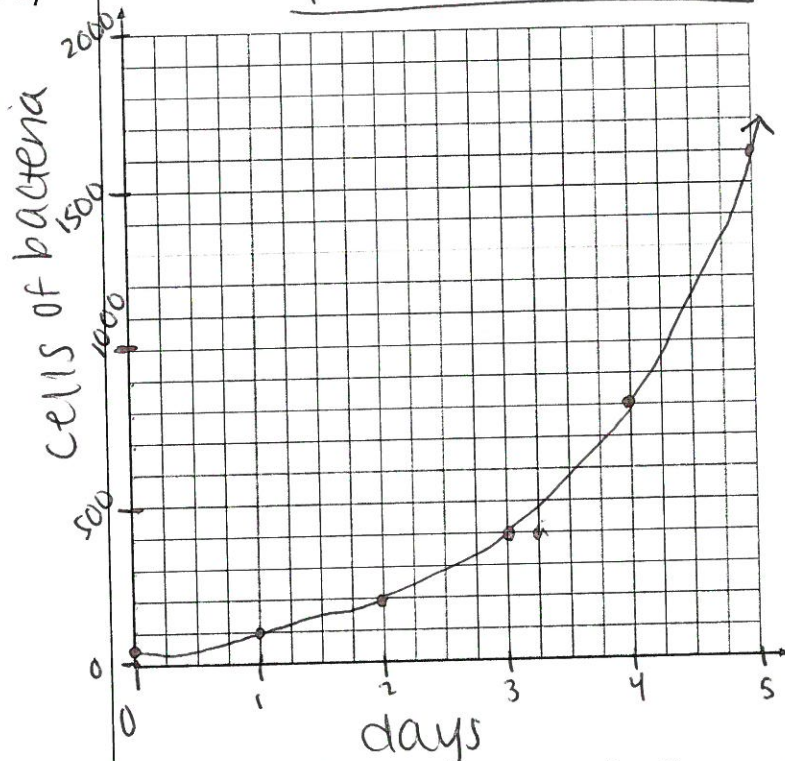
What is the initial value?  
50 cells of bacteria

What is the common ratio?  
 $\cdot 2$

Example  
(continued)

Graph the bacteria's growth over the five days.

Bacteria Growth



Fill in the table below to write the explicit rule for this pattern:

Day	Calculation	Cells
0	$50 \cdot 2^0 = 50 \cdot 1 = 50$	50
1	$50 \cdot 2^1 = 50 \cdot 2 = 100$	100
2	$50 \cdot 2^2 = 50 \cdot 4 = 200$	200
3	$50 \cdot 2^3 = 50 \cdot 8 = 400$	400
4	$50 \cdot 2^4 = 50 \cdot 16 = 800$	800
5	$50 \cdot 2^5 = 50 \cdot 32 = 1600$	1600
n	$50 \cdot 2^n = $ <del>50 \cdot 2^n</del>	$50 \cdot 2^n$

**Explicit Formula**

$f(n) = f(0) \cdot r^n$

$f(n)$  =  
output  
(y-value)

$r$  =  
common  
ratio

$n$  = term  
#

$f(0)$  = initial  
value  
(starting  
value)

<p><b>Exponents</b></p>	<p>Since the ratio is repeatedly multiplied, we use exponents to write the explicit rule. The ratio is multiplied <math>n</math> times because that is the term number. For example, in the problem above, if we wanted the number of bacteria on the 20<sup>th</sup> day, you would multiply 50 by 2 twenty times, or:</p> $50 \cdot 2^{20}$
<p><b>Using the Common Ratio</b></p>	<p>When we filled in the table for the bacteria, we were using the Recursive Rule again. However, since we multiply to get the next term, the recursive rule changes slightly:</p>
<p><b>Recursive Formula</b></p>	$f(n) = f(n-1) \cdot r; f(0) = a$ <div style="display: flex; justify-content: space-around; text-align: center;"> <div data-bbox="443 829 597 1018"> <math>f(n) =</math> current term's value         </div> <div data-bbox="722 829 941 1018"> <math>f(n-1) =</math> previous term's value         </div> <div data-bbox="1063 829 1258 966"> <math>r =</math> common ratio         </div> <div data-bbox="1291 829 1461 1018"> <math>f(0) = a</math> initial value         </div> </div>
<p>Example</p>	<p>For the bacteria's growth, the recursive formula would be:</p> $f(n) = f(n-1) \cdot 2 ; f(0) = 50$
<p>Example</p>	<p>Oxygen-14 is a radioactive isotope with a half-life of about 1 minute (every minute, half of it disappears). You have 10 pounds of Oxygen-14 to start with.</p> <p>a. What is the initial weight of the Oxygen-1<sup>4</sup>? What is the common ratio?</p> <p>initial weight <math>f(0) = 10</math> lbs Oxygen-14  common ratio <math>r = \div 2 \cdot 0.5 \cdot \frac{1}{2}</math></p>

Example  
(continued)

b. How much Oxygen-14 is left after 1 minute? 2, 3, 4, 5 minutes? Fill in the table.

Minutes	0	1	2	3	4	5
Pounds of Oxygen-14	10	5	2.5	1.25	0.625	0.3125

$\div 2$        $\div 2$        $\div 2$        $\div 2$        $\div 2$

c. What is the explicit formula for the sequence?

$$f(n) = f(0) \cdot r^n$$

$$f(n) = 10 (0.5)^n \quad \text{or} \quad f(n) = 10 \left(\frac{1}{2}\right)^n$$

d. What is the recursive formula for the sequence?

$$f(n) = f(n-1) \cdot r ; f(0) = a$$

$$f(n) = f(n-1) \cdot \frac{1}{2} ; f(0) = 10$$

$$f(n) = f(n-1) \cdot 0.5 ; f(0) = 10$$

$$f(n) = \frac{f(n-1)}{2} ; f(0) = 10$$