BHS Beaverton High School Oregon	**Topic/Objective: AGS 2 Module 3.9**	**Name:** Key
	Imaginary Numbers	**Period:**
		Date:

Essential Question: Describe the solutions to a quadratic that does not intersect the x-axis.

Questions:

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Notes:

$\mathbb{R}$

Big Idea #1: The set of **REAL** numbers contains all of the rational and irrational numbers.

Examples of **rational numbers**: whole #s, #s we can write as a ratio (fraction) of whole #s

$3, \frac{2}{3}, 7.8$

$\frac{3}{1} \quad \frac{2}{3} \quad \frac{39}{5}$

Examples of **irrational numbers**: many roots, π

$\sqrt{2} \quad \sqrt{5} \quad \sqrt{11}$

Solve using the quadratic formula: $y = x^2 - 6x + 13$

$a = 1$

$b = -6$

$c = 13$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(13)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{-16}}{2}$$

$$x = \frac{\cancel{6}^{3} \pm \cancel{4}^{2}i}{\cancel{2}_{1}} = \boxed{3 \pm 2i}$$

$\sqrt{-16} = 4i$

$-1 \quad 16$

$2 \quad 8$

$2 \quad 4$

$2 \quad 2$

What do you notice about the solution?

there is an imaginary number in the solution.

"Oiler"

A mathematician named Euler defined a new number: i = $\sqrt{-1}$. This is an **imaginary number**.

$i^2 = \sqrt{-1}$

i = $\sqrt{-1}$ i² = -1

Simplify:

1. $\sqrt{-25} = i\sqrt{25} = \boxed{5i}$

5 5

2. $\sqrt{-64} = i\sqrt{64}$ $\boxed{=8i}$

8 8

3. $\sqrt{-50} = i\sqrt{50} = \boxed{5i\sqrt{2}}$

2 25

5 $\cancel{5}$

4. $\sqrt{-8} = i\sqrt{8} = \boxed{2i\sqrt{2}}$

2 4

$\cancel{2}$ 2

5. $-3\sqrt{150} = -3 \cdot 5\sqrt{2 \cdot 3}$

$= \boxed{-15\sqrt{6}}$

2 75

3 25

5 $\cancel{5}$

6. $\sqrt{24} = \boxed{2\sqrt{6}}$

2 12

$\cancel{2}$ 6

2 3

7. $\sqrt{-20} = i\sqrt{20}$

2 10

$\cancel{2}$ 5

$= \boxed{2i\sqrt{5}}$

8. $\sqrt{-60} = i\sqrt{60}$

2 30

$\cancel{2}$ 15

3 5

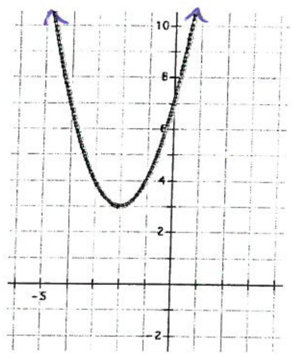
$= \boxed{2i\sqrt{15}}$

Big Idea #2: All **complex** numbers have a **real** and an **imaginary** part. They are written as $a \pm bi$.

real imaginary

Solve each of these by finding the x-intercepts.

9. $y = (x+2)^2 + 3$



$0 = (x+2)^2 + \cancel{3}$

$-3 \qquad \cancel{-3}$

$\pm\sqrt{-3} = \sqrt{(x+2)^2}$

$\pm\sqrt{-3} = x + \cancel{2}$

$-2 \qquad \cancel{-2}$

$x = -2 \pm \sqrt{-3}$

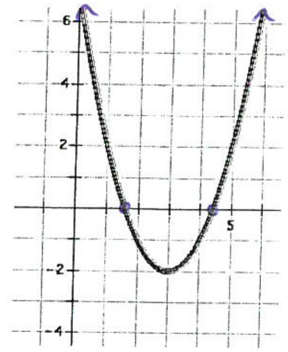
$\boxed{x = -2 \pm i\sqrt{3}}$

rational (real) imaginary

complex

Cannot put in calculator "error"

10. $y = (x-3)^2 - 2$



$0 = (x-3)^2 - \cancel{2}$

$+2 \qquad \cancel{+2}$

$\pm\sqrt{2} = \sqrt{(x-3)^2}$

$\pm\sqrt{2} = x - \cancel{3}$

$+3 \qquad \cancel{+3}$

$\boxed{x = 3 \pm \sqrt{2}}$

rational irrational

real

Can put in calculator

Which one has an imaginary solution? How can you determine this from the graph? #9 has an imaginary solution. we can see this from the graph because it doesn't cross the x-axis.

CTS sqrt method	Solve: 11. $x^2 - 4x + 10 = 0$ $0 = \underbrace{x^2 - 4x + \underline{4}} + 10 - \underline{4}$ $\quad \left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$ $0 = (x-2)^2 + \cancel{6}$ $-6 \qquad -\cancel{6}$ $(x + \frac{b}{2})^2$ $\pm\sqrt{-6} = \sqrt{(x-2)^2}$ $\pm i\sqrt{6} = x - \cancel{2}$ $+2 \qquad \cancel{+2}$ $\boxed{2 \pm i\sqrt{6} = x}$ $a=1$ $b=-4$ $c=10$ $x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(10)}}{2(1)}$ $x = \frac{4 \pm \sqrt{16-40}}{2}$ $x = \frac{4 \pm \sqrt{-24}}{2}$ $x = \frac{\cancel{4} \pm \cancel{2}i\sqrt{6}}{\cancel{2}}$ $\boxed{x = 2 \pm i\sqrt{6}}$ $\sqrt{24}$: 2, 12, 2, 6, 2, 3 12. $\cancel{-x^2} + \cancel{8x} = 20 + x^2 - 8x$ $\cancel{+x^2} \cancel{-8x}$ $0 = x^2 - 8x + 20$ $0 = \underbrace{x^2 - 8x + \underline{16}} + 20 - \underline{16}$ $\quad \left(\frac{-8}{2}\right)^2 = (-4)^2 = 16$ $0 = (x-4)^2 + 4$ $-4 \qquad -4$ $\pm\sqrt{-4} = \sqrt{(x-4)^2}$ $\pm 2i = x - 4$ $+4 \qquad +4$ $\boxed{4 \pm 2i = x}$ $a=1$ $b=-8$ $c=20$ $x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(20)}}{2(1)}$ $x = \frac{8 \pm \sqrt{-16}}{2}$ $x = \frac{\cancel{8} \pm \cancel{4}i}{\cancel{2}} = \boxed{4 \pm 2i}$

Summary: Describe the solutions to a quadratic that does not intersect the x-axis.

it must have an imaginary term!